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Anomalous Resonances and Relativistic Transparency for Ultra-high Intensity Laser Plasmas

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Abstract: **Relativistic oscillations are realized in Laboratory using intense ultrashort laser interacting with solid and nanostructure targets. Their study reveals various interesting aspects of laser plasma coupling at extremely high intensities. Conventional understanding shows that light goes beyond the critical density (n_c) due to relativistic mass effect the plasma transparency is enhanced and light goes up to $\gamma(t)n_c$. We find the transparency may be quite different and light may go up to $\gamma^3(t)n_c$ due to nonlinear resonance. It is further shown that non linear resonance in intense laser cluster and nanoparticle interaction is dynamically met due to time varying mass. This results in anomalous resonance densities which is different than the conventional value of resonance density of 3 n_c and 2n_c of spherical and cylindrical nanoparticles. Resonance are related to the time dependent Lorentz factor $\gamma(t)$ which must be evaluated numerically in realistic situations. However we obtain its approximate variation in different ways to see its effect in modifying the resonance density. For this the analogy of variable mass oscillators is borrowed which are not just for pedagogical interest but their study is relevant to understanding the complex phenomenon of relativistic oscillations. Based on the understanding of variable mass oscillators it is noted that time varying mass acts as an effective damping which increases the resonance density. This analytical study will give meaningful insight to help in choosing the nanostructure oscillators /targets and intense laser parameters to achieve resonance at high density and consequently efficient hot electron, ion and x-ray generation.**

Keyword: **Ultrashort Lasers, Laser Plasma Interaction, Relativistic Oscillator, Driven Oscillators, Nanostructures, Gas Clusters, Non linear interaction, Damping in light matter interaction, Linear and non linear resonance.**

I. INTRODUCTION

Intense ultrashort laser focussed on to a solid target results in the burst of X-rays, ions and hot electrons [1]. This radiation and particle source is also having the properties of being ultra short and point source [2]. Since brightness and flux of the consequent radiation and particle emission is quite high it is feasible for various applications like radiography [3], medical diagnosis and cure, inertial confinement fusion [4], time resolved studies of probing extreme dense hydro dynamically evolving matter etc. With the advent of Chirped Pulse Amplification techniques laser intensities can reach 10^{22}W/cm^2. The relativistic effects become increasingly important when the laser intensity exceeds 10^{18}W/cm^2 where the momentum of electron (mv) starts exceeding the rest mass momentum (m_0c) [4,5]. Apart from the usual application of multi TW or PW laser systems for electron acceleration or ion accelerators (compact table top) the high intensity lasers also enables one to do laboratory scale astro physics, particle physics by creating such equivalent condition over a micron scale lasting few femto-seconds [6,7] . These relativistic phenomena have been of great theoretical interest. One of them is the relativistic oscillator which has been pursued and still being studied analytically, numerically or through simulations [8,9]. The relativistic oscillator is highly nonlinear and finding its solutions is also quite interesting. It is also applicable for laser plasma interaction studies at relativistic intensities. One of the most direct consequence of relativistic effect is the beyond critical density (n_c) penetration [1,10] or the relativistic transparency. At intensities greater than 10^{18}W/cm^2 relativistic effects start dominating, the plasma frequency is lowered and the laser frequency matches the modified plasma frequency at γn_c. This effect of light penetration to higher density is also verified from the experimental results, analytical studies and simulations. We show the transparency is higher than the general simplistic perception and understanding. The enhanced transparency implies light may go up to $\gamma^3 n_c$ due to nonlinear resonance. Using the conventional approach of finding resonance density for relativistic laser planar solid interaction we show that the resonance density of nanostructure shows anomalous enhanced resonance density due to relativistic effect. Since relativistic driven oscillator problem cannot be solved analytically we correlate it with variable mass oscillator which is extensively studied for fundamental physics demonstration and comprehension [11-13]. Relativistic oscillator has a variable mass during the oscillation therefore similar analogy as a variable mass oscillator problem can be incorporated to perceive the consequences of variable mass on resonance of laser matter interaction. We introduce the role of time varying mass as an effective damping which increases the resonance density and therefore anomalous high resonance densities can be encountered in laser interaction of matter at ultrahigh intensities. The paper is organised as follows; first we mention the popular variable mass oscillator problem, then briefly describe the well known phenomenon of beyond critical density penetration at relativistic intensity in planar solid. This is followed by showing that the expected resonance density of nanostructure is anomalous at high intensity from a very simple consideration. Then we discuss the non linear resonance of laser solid and nanostructure interaction and prove that resonance density is actually $\gamma^3(t)$ using the relativistically driven oscillator model. We remark that the time dependent Lorentz factor must be evaluated for estimating the non linear resonance of planar and nanostructures. As a simple way to understand this formidable problem to explain the cause of upshift of resonance density in relativistic laser matter interaction, the variable mass oscillator analogy is applied.

II. ANALYTICAL METHODS TO STUDY VARIABLE MASS SYSTEMS

A. Variable mass oscillators

The variable mass oscillator comprise of many situations like a leaky pendulum, a leaky mass suspended from spring etc [11-13]. A vast amount of literature available shows various experiments and analytical ways to study this insightful problem. Relativistic oscillator is also variable mass problem where during the oscillation mass keeps on changing. Fig. 1 shows various examples of variable mass oscillators and that these kinds of oscillations are regularly encountered in Laser matter interaction studies at ultrahigh intensities.

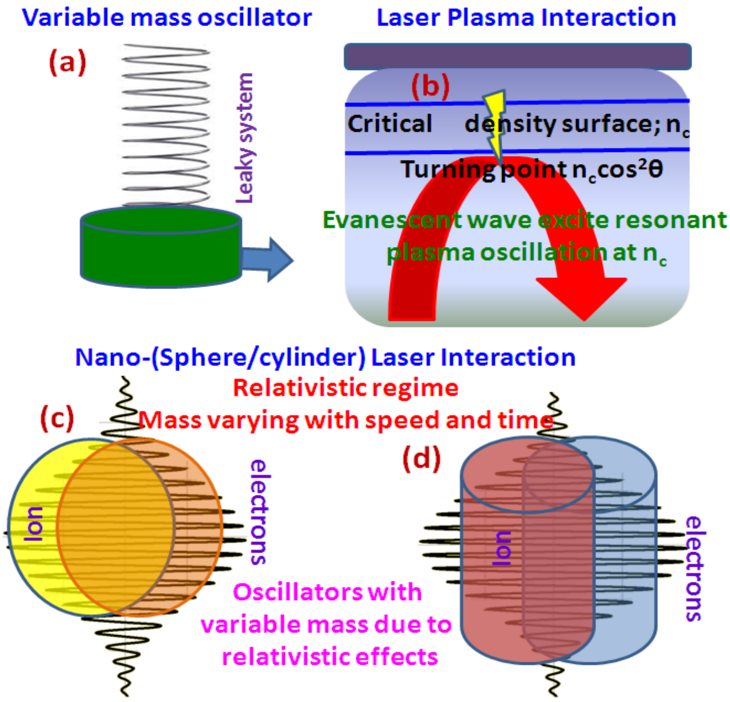

Fig. (1) All the figures are example of variable mass oscillator. (a) A variable mass oscillator whose natural frequency keeps on changing due to leaking mass (b) Laser plasma interaction for solid target where evanescent laser field from the turning point $n_c\cos^2\theta$ excites resonant oscillations at n_c. At relativistic intensities light goes beyond the critical density(c)

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Laser interacting with spherical nanoparticle like gas clusters, (d) Laser interacting with cylindrical nanoparticle like nanorod. At relativistic intensities the electron cloud mass keeps on changing due to relativistic effects and the resonance density is expected to change as compared to the generally known non relativistic resonances.

B. Beyond critical density penetration (Relativistic effect)

Light propagation in plasmas is understood by dielectric constant of plasma as shown in eq. 1 (ε depends on laser frequency ω and plasma frequency ω_p or alternatively on the electron density n_e and the critical density n_c given in eq. 2). ε becomes zero at resonance. Physically this means that the polarisation reversal occurs around resonance. Planar solid has spatially decreasing plasma density and the laser goes up to the critical density beyond which the propagation vector becomes imaginary. However relativistic laser goes beyond n_c since the plasma frequency is expected to decrease (eq. 3).

$$\varepsilon = 1 - \frac{\omega_p^{\,2}}{\omega^2} \quad ; \quad \varepsilon = 1 - \frac{n_e}{n_c} \qquad (1)$$

$$n_c = \frac{4\pi^2 c^2 \varepsilon_0 m_0}{e^2 \lambda^2}; \; n_c = \frac{1.1\mathrm{x}10^{21} / cc}{\lambda(\mu m)^2} \qquad (2)$$

$$\omega_p^{\,2} = \frac{n_e e^2}{m\varepsilon_0} \quad ; m = \gamma \mathrm{m}_0 \quad ; \gamma = \frac{1}{(1 - v^2 / c^2)^{1/2}} \qquad (3)$$

At high intensity mass becomes relativistic and light goes up to γn_c as shown in eq.4. The Lorentz factor γ is generally calculated from the value of the laser strength parameter a_0 given in eq. (5). It is important to caution that the laser strength parameter which is defined as maximum momentum of free electron oscillating in cosine field. In plasmas electrons are oscillating around fixed ion core at ω_p. So they are not exactly free in the strict sense of the word. This aspect will be explored later in section D, E and F. γ is always greater than 1 as seen from its relation with a_0 from eq. (6). Therefore depending on the intensity the light can penetrate up to γn_c as shown in Fig 2 (a) plotted using eq. 7 and 8.

$$\frac{n_e}{n_c} = \gamma > 1 \quad \text{Beyond critical density penetration} \qquad (4)$$

$$\mathrm{a}_0 = \frac{\mathrm{qE}_0}{m_0 \omega c} \; = 0.85\mathrm{x}\lambda(\mu m)\mathrm{x}\sqrt{I_{18}(W / cm^2)} \qquad (5)$$

$$\gamma^2 = a_0^{\,2} + 1 \qquad (6)$$

C. Method I: Resonance density at relativistic intensities (Anomalous densities for nanostructures; γ found from I directly without solving the relativistic equation of motion)

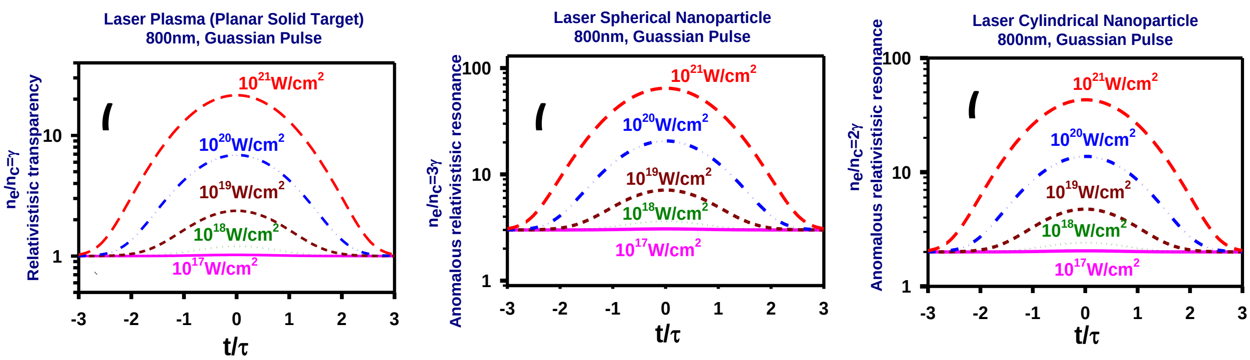

Fig. 2 Resonance density plots dependence on laser intensity (800nm Ti: Sa laser) of planar solid target assuming Lorentz factor γ calculated directly from Laser intensity using equation 8. (a) Planar solid (b) Spherical (c) Cylindrical Nanoparticle. Graphs look similar but their values(Y axis) are different.

Applying the same convention that at high intensity plasma frequency ω_p decreases the resonance density of spherical and cylindrical nanoparticle can be found. The electric field inside a spherical and cylindrical dielectric is given by $E_{in}=3E_0/(\varepsilon+2)$ for nano sphere/gas clusters [14] and $E_{in}=2E_0/(\varepsilon+1)$ for nanocylinder [15-18]. Using eq. 1 and 3 the anomalous resonance density is readily obtained as $3\gamma n_c$ for sphere and $2\gamma n_c$ Cylinder. Again depending on the intensity (Gaussian pulse @ 800 nm) the anomalous resonance densities are shown in Fig 2 (a) and (b) plotted using eq. 7 and 8. From these figures it is clear that in order to excite resonance at high densities high laser intensity laser in the relativistic regime is quite useful. Cluster expansion and longer laser pulses are not required and just by the tuning of laser intensity resonance can be excited even up to solid densities ($\approx 50n_c$). Fig. (2) is plotted based on the equations derived from γ(t) obtained from the laser intensity directly. However the correct procedure should be to solve the relativistic laser driven oscillator and finding velocity v (t) from its equation of motion and then find $\gamma(t)=1/(1-v(t)^2/c^2)^{1/2}$.

$$I=I_0 e^{-\frac{t^2}{\tau^2}} \text{ Guassian pulse } (\lambda=800\text{nm}) \tag{7}$$

$$\begin{bmatrix} \left(\dfrac{n_e}{n_c}\right)_{Planar} = \gamma\ (\text{t}) = \left(0.4624\text{x}I_{18}(t)+1\right)^{1/2} \\ \left(\dfrac{n_e}{n_c}\right)_{Sphere\ \ \mathrm{Res}} = 3\gamma\ (\text{t}) = 3\left(0.4624\text{x}I_{18}(t)+1\right)^{1/2} \\ \left(\dfrac{n_e}{n_c}\right)_{Cylinder\ \ \mathrm{Res}} = 2\gamma\ (\text{t}) = 2\left(0.4624\text{x}I_{18}(t)+1\right)^{1/2} \end{bmatrix} \tag{8}$$

D. Relativistic harmonic oscillator

To find the exact anomalous resonance density in nanostructures and relativistic transparency in laser planar solid target plasmas relativistic driven oscillator Hamiltonian is written [19] using eq.(9) and from this the velocity and equation of motion is obtained from eq. (10) and (11).

Hamiltonian of relativistic oscilation $H=\sqrt{p^2c^2+m_0{}^2c^4}+ky^2/2+yqE_0\cos\omega t$ (9)

Velocity obtained from Hamiltonian $\dfrac{dy}{dt}=\dfrac{pc^2}{\sqrt{p^2c^2+m_0{}^2c^4}}$ (10)

Eq of motion of driven oscilator $\dfrac{dp}{dt}+ky=qE_0\cos\omega t$; $(p=\gamma m_0 v)$ (11)

It is important to note here that mass m is t dependent and rate of change of p (dp/dt) should be carefully calculated

after simplification of eq (11) $\dfrac{m_0\ddot{y}}{(1-v^2/c^2)^{3/2}}+ky=qE_0\cos\omega t$

$$\ddot{y}+(1-v^2/c^2)^{3/2}\frac{ky}{m_o}=\frac{qE_0\cos\omega t}{m_0}(1-v^2/c^2)^{3/2}$$

$$\ddot{y}+\frac{ky}{\gamma^3(t)m_o}=\frac{qE_0\cos\omega t}{m_0\gamma^3(t)} \tag{12}$$

γ should not be directly calculated from Intensity I but after finding v from equation of motion

Eq. (12) which can only be solved numerically but some interpretations can be made from it. Eq. (12) can be reduced to an equivalent simple harmonic oscillator having a time dependent oscillation frequency ω_{eff} (t) driven by a time varying force F (t) as shown in eq. (13). F (t) can be decomposed in to various Fourier components of (ω) the most basic non linear resonance condition is given eq. (14). This analysis gives the correct anomalous resonance density.

Effective equation of motion $\ddot{y}+\omega_{eff}{}^2(t)y=F(t)$ (13)

Non linear resonance condition $\omega_{eff}(t)=\omega$ (Resonance condition) (14)

E. Method II: Resonance density at relativistic intensities ($\gamma=1/(1-v^2/c^2)^{1/2}$; v found from equation of motion)

From the above presented elaborate analysis of relativistic harmonic oscillator driven by a cosine field the non linear resonance condition is obtained. This condition is highly dynamical since ω_{eff} (t) keeps on varying with time. In section B and C the value of γ was calculated directly from intensity however the correct procedure will be to find it using the basic relation $\gamma=1/(1-v^2/c^2)^{1/2}$ where v found from equation of motion given from eq.(12) or (13). Therefore the resonance density for planar solid, spherical and cylindrical nanoparticle is given from the values as shown in eq.(15). The dynamical value $\gamma^3(t)$ is not necessarily very large as expected from the analysis in section B and C. Rather its value must be obtained numerically. In the next section a simplistic approach is presented that the laser can penetrate even beyond γ n_c. The exact and anomalous transparency cannot be obtained analytically however the variable mass oscillator analogy enlightens that the plasmas can be more transparent than we usually think.

$$\left[\left(\frac{n_e}{n_c}\right)_{Planar\ Res} = \gamma^3(\mathrm{t})\ ;\ \left(\frac{n_e}{n_c}\right)_{Sphere\ \ Res} = 3\gamma^3(\mathrm{t});\ \left(\frac{n_e}{n_c}\right)_{Cylinder\ Res} = 2\gamma^3(\mathrm{t})\right] \qquad (15)$$

F. *Time varying mass as effective damping*

From the variable mass oscillator principles [11-13] we can expanded Eq. (12) in the form as shown in eq. (16) and rearranged as eq. (17). These oscillators are studied with the motive of finding the effect of time varying mass on Time period or frequency of the resultant motion. Eq. (17) has a close resemblance with a damped driven oscillator where time varying mass is equivalent to damping (the term (1/m)(dm/dt)). It must also be noted from eq. (17) that the R.H.S. is not a pure cosine function as time dependent m is also appearing. But the Fourier components of a general time dependent function can help solve this equation approximately.

$$m\frac{dv}{dt} + v\frac{dm}{dt} + ky = qE_0\cos\omega t \qquad (16)$$

$$\frac{dv}{dt} + v\left(\frac{1}{m}\frac{dm}{dt}\right) + \frac{k}{m}y = \frac{qE_0}{m}\cos\omega t \qquad (17)$$

Eq. (18) is the damped driven oscillator differential equation whose solution is given in eq.(19) and eq.(20).

$$\ddot{y} + 2\beta\dot{y} + \Omega^2 y = f_0\cos\omega t \qquad (18)$$

$$y(t) = \frac{f_0\cos(\omega t - \phi)}{\sqrt{(\Omega^2-\omega^2)^2 - 4\beta^2\omega^2}} \qquad (19)$$

$$\tan\phi = \frac{2\beta\omega}{(\Omega^2-\omega^2)} \qquad (20)$$

The resonance frequency is generally independent of damping if its magnitude is small but in case the damping is prominent it leads to the modification of resonance frequency. Eq. (21) shows the decrease of natural resonance frequency Ω due to the damping term. The damping term is arising due to the time varying mass given from eq. (22)

$$\omega = \sqrt{\Omega^2 - 2\beta^2} \qquad (21)$$

$$2\beta = \frac{1}{m}\frac{dm}{dt};\ \ \Omega^2 = \frac{k}{m}\ ;\ f_0 = \frac{qE_0}{m} \qquad (22)$$

G. *Method III: Resonance density increase due to damping because of the time varying mass (damping found from free electron motion in relativistic field)*

From the section F we can expect that the decrease of natural resonance frequency Ω due to damping will cause rise in the resonance density. The term (1/m)(dm/dt) has to be dynamically evaluated but for a first order approximation let us find this damping term for the case of a free electron in a relativistic field. This problem is exactly solvable and can give an idea how the damping evolves in time. Eq. (23) shows the resonance condition that can be written in several forms.

$$\omega = \sqrt{\Omega^2 - \frac{1}{2}\left(\frac{1}{m}\frac{dm}{dt}\right)^2} ; \omega = \sqrt{\frac{k}{\gamma m_0} - \frac{1}{2}\left(\frac{\dot{\gamma}}{\gamma}\right)^2} ; \omega = \sqrt{\frac{k}{\gamma m_0} - \frac{1}{2}\left(\frac{d\ln(\gamma)}{dt}\right)^2} \qquad (23)$$

The equation of motion of free relativistic electron is given in eq.(24). From the solution of this equation time dependent values of β (v/c) and γ can be easily obtained as shown in eq. (25) and (26).

$$\frac{dv}{dt} + v\left(\frac{1}{m}\frac{dm}{dt}\right) = \frac{qE_0}{m}\cos\omega t \qquad (24)$$

$$\frac{\beta}{\sqrt{1-\beta^2}} = a_0 \sin\omega t; \ \beta = \frac{a_0 \sin\omega t}{\sqrt{a_0^{\,2}\sin^2\omega t + 1}} \qquad (25)$$

$$\gamma = \sqrt{a_0^{\,2}\sin^2\omega t + 1} \qquad (26)$$

The resonance frequency from eq. (23) require the evaluation of the term d/dt(ln(γ)) , whose time dependence is shown in eq. (27) The resonant condition thus becomes a highly dynamical time dependent quantity shown in eq. (28)

$$\frac{\dot{\gamma}}{\gamma} = \frac{\omega a_0^{\,2}\sin(2\omega t)}{a_0^{\,2}\sin^2(\omega t) + 1} \qquad (27)$$

$$\omega^2 = \omega(t)^2 \approx \frac{k}{\sqrt{a_0^{\,2} + 1}m_0} - \frac{1}{2}\left(\frac{\omega a_0^{\,2}\sin(2\omega t)}{a_0^{\,2}\sin^2(\omega t) + 1}\right)^2 \qquad (28)$$

The resonance density for planar solid plasmas, spherical and cylindrical nanoparticles is easily obtained from eq. (28) and shown in eq.(29)

$$\frac{n_e}{n_c} \approx G\sqrt{a_0^{\,2} + 1}\left(1 + \frac{1}{8}\left(\frac{a_0^{\,2}\sin(2\omega t)}{a_0^{\,2}\sin^2(\omega t) + 1}\right)^2\right) \quad \text{(Resonance condition)} \qquad (29)$$

G=1 Planar ; G=3 Nano-Sphere; and G =2 for Nano-cylinder

This simple analysis clearly shows that anomalous resonance densities can be even higher and quite dynamical when time varying mass in relativistic domain is properly considered.

III. CONCLUSION

In conclusion a careful note must be taken regarding the common perception that ultrahigh intensity light goes up to $\gamma(t)n_c$ owing to the relativistic mass effect. In exact terms this assumption has to be modified since the transparency is higher and quite dynamical at higher intensity. It is due to nonlinear resonance that anomalous transparency is expected where light may go up to $\gamma^3(t)n_c$. Resonance density dependence on laser intensity of planar solid, spherical and cylindrical nanoparticle targets are derived, assuming Lorentz factor γ is calculated directly from Laser intensity. Instead of the resonance density of $3n_c$ and 2 n_c for spherical and cylindrical nanoparticles at non relativistic intensities the resonance is boosted by the Lorentz factor γ at high intensities. It is further shown that nonlinear resonance in intense laser cluster and nanoparticle interaction is dynamically excited due to time varying mass and resonance density may be further enhanced. The interdisciplinary phenomenon of variable mass oscillators enlightens some of the aspects of relativistic oscillation where mass becomes velocity and time dependent. We also introduce the role of time varying mass as an effective damping which increases the resonance density. Solving the relativistic driven harmonic oscillator fully analytically is an uphill task but using simple arguments based on logic and reason (based on the variable mass oscillator studies), the physical processes occurring during the dynamics can be understood. This analytical study will help in laser energy coupling at relativistic intensities to excite non linear resonance at high density that will help in producing copious hot electron, ion and x-ray generation.

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